GRAPHICAL TREATMENT
OF THE
INDUCTION MOTOR

HEYLAND

A GRAPHICAL TREATMENT

OF THE

INDUCTION MOTOR

BY

ALEXANDER HEYLAND

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G. H. ROWE and R. E. HELLMUND

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A GRAPHICAL TREATMENT OF THE INDUCTION MOTOR.

The object of the method described in the following pages is the experimental determination of the characteristic properties of induction motors. It consists essentially in the practical application of the circle diagram, first described by the writer in 1894.* It is based on two simple and quickly performed experimental tests on the finished motor.

The method shows at a glance the main properties of a motor and its commercial excellence. The writer has used the circle diagram several years in the testing room for comparing calculated values with results obtained from tests, and it has well served its purpose.

Before describing the method and its applications, perhaps I may be allowed to present briefly the theory of the induction motor, and the derivation of the diagram.

General Theory of the Induction Motor.

The induction motor is in principle a transformer. The exciting member *A* (Fig. 1) represents the inducing or primary circuit; the short-circuited member *B* repre-

*Elektrotechnische Zeitschrift, Oct. 11, 1894, p. 561.

sents the induced or secondary circuit. The alternating fields produced by the current in the exciting coils combine in the well known manner to form a rotating field. The motor is, therefore, a *transformer with a rotating field*, and the load of the system is determined at any time by the difference between the constant speed of rotation of the field produced in the stationary member A, and that of the rotating short-circuited member B. The turning of the rotor is due

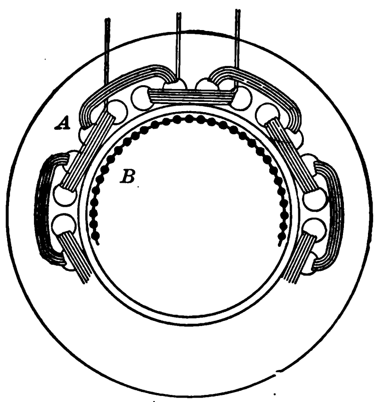


Fig. 1.

to the torque existing between the rotating field and the currents produced in the short-circuited member by rotation in the field. These currents, and therefore also the torque, vary directly with the product of the slip and the field interlinked with the secondary conductors.

The slip is defined as the difference between the speed of rotation of the exciting field, and that of the short-circuited secondary. The variation of the slip in

the induction motor has the same significance as the variation of load of the ordinary transformer, due to a change of its external resistance.

With constant impressed electromotive force, the field produced by the current in the primary winding is constant for all loads, as in the transformer. This, however, is not true in the short-circuited member, which is the real work transmitting element. Herein lies the essential difference between the induction motor and the transformer without leakage, in which, as is well known, the total primary field passes through the secondary. Since there must, of necessity, be an air-gap between the short circuited windings and the primary windings, not all of the magnetic lines ϕ induced in the primary member pass through into the secondary. But a part ϕ_s passes directly through the space between the two windings back into the primary member, and only the *difference $\phi_A = \phi - \phi_s$ passes into the short-circuited member. This field ϕ_A produces, in the latter, currents which lag, according to the law of induction, a quarter period behind the field, *i.e.*, they are in quadrature with the inducing field, and therefore the torque produced is proportional to the product of induced currents and primary field.

The lines ϕ_s are called leakage lines or leakage field, and since they are caused by the primary current I_1

*In fact there is not only a primary leakage, but also a secondary leakage that is a flux which is interlinked with the secondary conductors only. It is however admirable, and simplifies further considerations, to assume that the primary and secondary leakage fluxes are combined to form one single leakage flux—i. e. the primary leakage flux ϕ_s. The inexactness introduced thereby is of no practical importance in the results obtained from the following derivation.—ROWE HELLMUND.

only, they must be proportional to it. If we designate the reluctance of the leakage path by ρ_s, then we have

$$\phi_s \propto \frac{I_1}{\rho_s}$$

The main field, as in the transformer, is of constant amplitude $= \phi$. Therefore, the armature field ϕ_A is given by the difference between the main field and the leakage field $\phi_A = \phi - \phi_s$ (Fig. 2), and is pro-

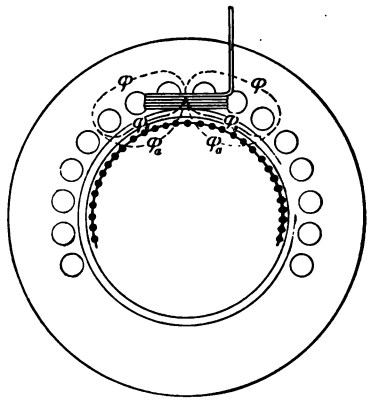


FIG. 2.

duced by the difference between the ampere turns of the primary and secondary circuits. If we call the resulting magnetizing current, I, and the reluctance of the path of the secondary field ρ, then

$$\phi_A \propto \frac{I}{\rho}$$

This magnetizing current is thus not constant, as in the transformer, but decreases with increasing load since with increasing load the current I_1 and therefore ϕ_s increases. ϕ_A will therefore decrease.

If no current is produced in the secondary member, the primary winding carries only the magnetizing current, and therefore the primary current equals the magnetizing current, or

$$I_1 = I$$

This occurs when the motor is running unloaded, and the slip is zero.

As the load increases, we have the following phenomena: The slip increases, and with it the electromotive force induced in the secondary member, and the current I_2 resulting therefrom increases. This secondary current I_2 exerts a demagnetizing effect on the field, and therefore causes, as in the transformer, a corresponding increase of the current in the primary member. The demagnetizing influence of the current I_2 acts only on the armature field. The strength of the leakage field increases, however, in direct ratio with the increase of the primary current. Therefore, the increase of the load, and the increase of the secondary current, have the following consequences:

(1) The secondary field is decreased by the demagnetizing action of I_2.

(2) Indirectly, the leakage field is increased, due to an increase of I_1.

Thus, the leakage field is directly proportional to I_1. The armature field equals the difference between the main and leakage fields, and therefore the sum of the armature field and leakage field must be equal to the main field and constant.

DIAGRAM OF THE INDUCTION MOTOR.

As in the well known transformer diagram, the primary current, and the secondary current in the short-circuited secondary, can be represented by two sides

of a triangle (Fig. 3), of which the resulting third side is the magnetizing current I.

This magnetizing current produces the secondary field, which interlinks with both the exciting and short-circuited windings. The secondary field produces the currents in the short-circuited secondary; it being in fact the only field which passes into the secondary. The secondary currents, therefore, lag exactly a quarter period behind the secondary field. As shown before, the main difference between the transformer without leakage and the induction motor, lies in the fact that

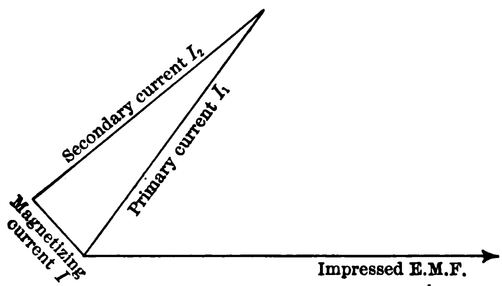


FIG. 3.

the magnitude of the magnetizing current changes, and that the current triangle changes its position.

We know (Fig. 3) that the constant main field, which induces a counter electromotive force corresponding to the impressed electromotive force is at right angles to the impressed electromotive force in a circuit without resistance, and without iron losses. Under these conditions, in a transformer without leakage, the magnetizing current is therefore at right angles to the impressed electromotive force, and is constant.

Further, the current triangle always has the same relative position to the electromotive force.

In the induction motor, however, the main field must be resolved into two parts. One part, the leakage field, is in phase with the primary current, and proportional to it. The other part, the secondary field, must therefore be variable, and cannot be in phase with the main field. The magnetizing current which produces this secondary field, must therefore vary in amplitude and direction. It is possible to follow these changes by the graphical combination of the fields (Fig. 4).

The diagram proposed by the writer is based on

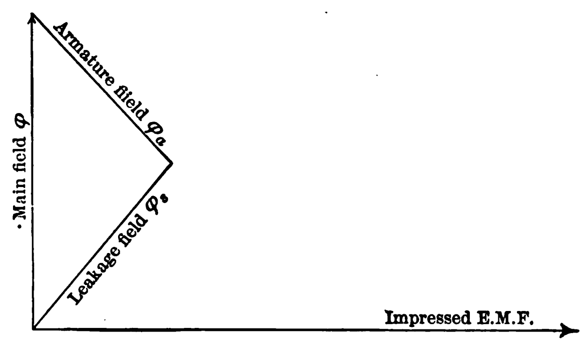


FIG. 4.

these assumptions, and is represented in Fig. 5.

The leakage field is directly proportional to the primary current, and in phase with it, or

$$\phi_s \propto \frac{I_1}{\rho_s} = \mathrm{A}\,C'$$

Thus, we can represent the leakage field according to a definite scale by the vector A C′ $= I_1$. The main field may be represented by a line A D, constant in

magnitude and in quadrature with the electromotive force.

$$\phi \propto A\,D$$

The third side $C'\,D$, must therefore give the secondary field, or

$$\phi_A = \frac{I}{\rho} = C'\,D$$

and $C'\,D$ must be in quadrature with $c'\,C' = I_2$, and

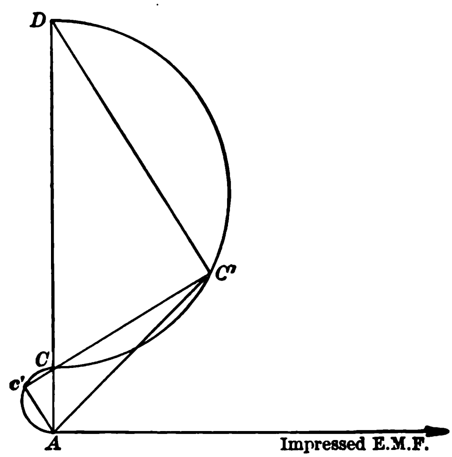


FIG. 5.

parallel with $A\,c' = I$. Further, there must be a constant relation between $C'\,D$ and $A\,c'$ according to an equation previously given, or

$$\frac{C'\,D}{A\,c'} = \frac{\phi_a}{I} \propto \frac{I}{\rho}$$

In order to satisfy this equation when the load changes, the points c' and C' must move on the two

half circles represented in Fig. 5. The angles $A\,c'\,C$ and $C\,C'\,D$ are always right angles, and the ratio $\frac{C'\,D}{A\,c'}$ is constant.

We see from the figure that the current cannot increase indefinitely, even if we assume the resistance to be zero. It reaches a maximum when C' coincides with D, or

$$I_1 = A\,D.$$

The leakage field, which is then represented by the line $A\,D$, equals in intensity the main field, and the remaining part, the secondary field, becomes zero.

In this case

$$\phi = \phi_s \propto \frac{I_1}{\rho_s} \propto \frac{A\,D}{\rho_s}$$

Let us study now the condition for minimum current. The minimum current flows when the motor is running without load. In this case $I_2 = O$, and therefore $I_1 =$ the magnetizing current, or

$$I_1 = I = A\,C, \text{ and}$$

$$\phi \propto \frac{I}{\rho} \propto \frac{I_1}{\rho} \propto \frac{A\,C}{\rho}$$

or, since ϕ remains constant, we have the relation:

$$\frac{\text{Maximum current}}{\text{Minimum current}} = \frac{A\,D}{A\,C} = \frac{\rho_s}{\rho} =$$

$$\frac{\text{reluctance of leakage path}}{\text{reluctance of path of secondary field}}$$

The above considerations may be briefly stated as follows: In induction motors the relation between the

electromotive force and the current can be represented by a vector diagram. In this diagram, the vector representing the current, which, as we know, varies with the load, is determined by the fact that its end point moves in a circle; the position of which is given by the ratio

$$\frac{A\,D}{A\,C} = \frac{\rho_s}{\rho} = \frac{\text{reluctance of leakage path}}{\text{reluctance of path of secondary field}}$$

DETERMINATION OF INPUT, OUTPUT, TORQUE AND SLIP.

So far, we have investigated current magnitudes and phase differences only. To utilize the diagram, practically, and to determine from it the operation and commercial excellence of a motor, we must introduce the friction, iron losses, and especially the electrical losses.

The friction and iron losses can be considered constant, since the speed of the motor and the field between no load and full load are practically constant. The iron losses of the short-circuited secondary can be neglected on account of the low frequency of its field. We shall see later that the exciting field does not remain quite constant, but is somewhat reduced by the ohmic drop in the primary circuit. On the other hand, the iron losses in the secondary increase slightly with the load, due to increasing slip. These variations will almost neutralize each other, so that the total iron losses are approximately constant.

It is quite different with the copper losses. As is well known, these are proportional to the square of the current. For this reason, it is very difficult to represent them directly in the diagram. We shall see however, that their influence appears in another form,

viz., by weakening the field, whereby they may be very readily considered. The ohmic drop of the primary winding—$I_1 \times R_1$ (I_1 = current, and R_1 = ohmic resistance of primary circuit) opposes the primary impressed electromotive force, and consequently the field does not need to induce a counter electromotive force equal to the impressed, but one equal to the difference between the impressed electromotive force and the drop. It follows that the main field, inducing the counter-electromotive force, and, therefore, also the secondary field, will be decreased by an amount corresponding to the ohmic drop. The diagram was, however, constructed on the assumption of a constant main field. Yet the omission of the loss of potential, due to resistance, does not change the correctness of the diagram, since, instead of the drop, we can introduce an equivalent field, which may finally be subtracted from the resulting secondary field.

Since an electromotive force always corresponds to a field at right angles to itself, and is proportional to it, we may take the ohmic drop into consideration by a reduction of the field proportional to and at right angles to the drop, that is at right angles to the direction of the current. Thus, we arrive at correct results if we consider the main field constant in our diagram, and finally decrease the secondary field by an amount corresponding to the drop in potential.

In the diagram, the primary current $A\,C'$ may be resolved into two components, $A\,C$ and $C\,C'$, of which the no load component $A\,C$ causes a constant drop. The other component $C\,C'$ is variable, and causes a drop in potential which appears in the form of a diminution of the field ϕ_a proportional to and in quadrature with this component. In other words, since part of

th3 impressed electromotive force is used in overcoming the ohmic resistance, the secondary field needs only to be assumed smaller by an amount corresponding to the ohmic drop. Let us call this part to be subtracted

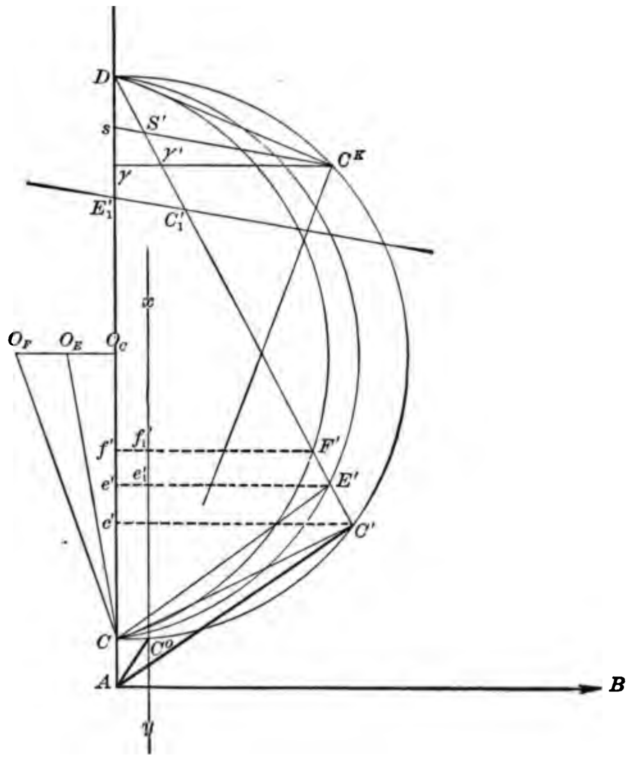


FIG. 6.

$C' E'$; then $C' E'$ is proportional to $C C'$; that is, the point E' must move on the circumference of the second circle shown in Fig. 6. This circle, therefore, indicates that the secondary field is not represented as before

by $D\,C'$, but by the line $D\,E'$ The main field decreases, of course, by the same amount. However, since we do not use the main field in further investigation, it is unnecessary to introduce the change in the diagram.

Similarly, the loss in the short-circuited secondary appears in the diagram as the line $E'\,F'$. Introducing in this way the ohmic drop of both windings by a corresponding reduction of the secondary field, we are in a position to derive the various quantities, such as torque, output, etc.

(a) The *electrical input* $= 3\,e\,i \cos\theta = \sqrt{3}\,E\,i\cos\theta$, can be found, since the impressed electromotive force is constant, by scaling the line $c'\,C'$, that is, the energy component of the primary current $A\,C'$,

$$\therefore \text{Input} \propto c'\,C'$$

(b) The *torque* is determined by the product of the secondary field and the secondary current, and is proportional to $E'\,D \times C'\,C$, *i.e.*, to the area of the triangle $C\,E'\,D$ (since $E'\,D$ is the base, and $C'\,C$ the altitude of this triangle). One side, $C\,D$, remaining constant, we can represent the area of the triangle by the altitude $E'\ e'$, or the torque is proportional to the abcissa of the point E'.

Now, there are a number of constant losses, such as those due to bearing and air friction, iron losses and copper losses at no load. These losses being constant, manifest themselves in the diagram by a constant diminution of the torque. In other words, the line representing the torque, $E'\ e'$, is for example, diminished by a length $e'\,e_1'$, proportional to the constant losses. Therefore, drawing a line parallel to the axis of Y, and

at a distance from it equal to $e' e_1'$, we find the actual torque represented by the line $e_1' E'$, or

$$\text{Torque} \propto e_1' E'$$

(c) The *output* is given by considering the loss in the short-circuited secondary $E' F'$ as the reduced altitude of the triangle $C F' D$, or

$$\text{Output} \propto f_1' F'$$

(d) The *slip* results from the fact that the currents produced in the secondary are proportional to the product of secondary field and the slip, or

$$\text{Slip} \propto \frac{\text{secondary current}}{\text{secondary field}} \propto \frac{C' C}{E' D}.$$

But $C' C \propto E' C$, and therefore, also, slip $\propto \dfrac{E' C}{E' D}$

Draw in the diagram any straight line $E_1' C_1'$, so that the angle $D E_1' C_1' = D E' C$. $D E' C$ is constant for all loads, since all angles inscribed in the same segment are equal, and we have

$$\frac{E' C}{E' D} = \frac{E_1' C_1'}{E_1' D}$$

In the above proportion $E_1' D$ remains constant; then

$$\frac{E' C}{E' D} \propto E_1' C_1'$$

Therefore, the slip is proportional to $E_1' C_1'$. Thus we have in $E_1' C_1'$ a measure of the slip.

In order to find the value of the slip, we may determine the point where the slip is 100%, that is when

the rotor is stationary. In this case the output must be zero, and the point F must coincide with D. This happens when C' coincides with C^k and the line $C^k D$ is tangent to the circle O_F. Draw the line $s\,C^k$ parallel to $E_1'\,C_1'$; then the intercept $s\,s'$ is proportional to $E_1'\,C_1'$, and is in direct ratio to $s\,C^k$ (100% slip) or slip $= s\,S'$.

Finally, the electrical efficiency may be determined. The electrical loss in per cent. of the input is proportional to

$$\frac{\text{secondary current}^2}{\text{Input}} \propto \frac{C'C^2}{c'\,C'} \propto \frac{C'\,C}{C'\,D} \text{[since } c'C \propto C'C \; x \; C'D]$$

Drawing a line $C^k\,\gamma$ from the point C^k at right angles to $A\,D$, we obtain directly the above ratio (as in the case of the slip) in the line $\gamma\,\gamma'$, and the electrical efficiency in the remainder of the line $\gamma\,C^k$, or

$$\text{Electrical Efficiency} = C^k\,\gamma'.$$

The diagram now gives us a complete determination of all the essential characteristics of an induction motor.

At no load (Fig. 7), the point C° is fully determined by the no load losses, viz., the iron losses, friction and windage, and the copper loss of the magnetizing current. These losses are represented by an energy current $C\,C^\circ$, which, combined with the magnetizing current $A\,C$ gives the no load current $A\,C^\circ$.

The torque and output are zero, and the slip very small. With increasing load, the point C' moves along the circumference of the circle O_c. The phase difference θ between the electromotive force and the current decreases, that is, the power factor increases. At the point C', where $A\,C'$ is tangent to the circle O_c, the

phase difference is a minimum. The torque is $e' E'$, the output $f' F'$, and the slip $s S'$.

The output increases with increasing slip, and reaches

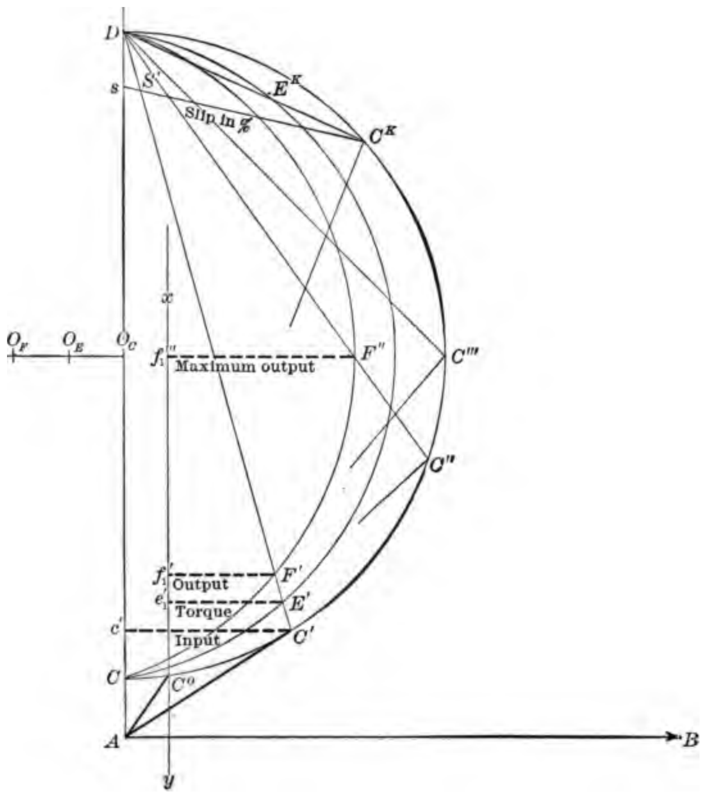


FIG. 7.

a maximum value at the point F''. If the slip still further increases, the output diminishes; the input still increases, but later also decreases, while the phase difference rapidly increases. The output becomes

equal to zero when the slip equals 100%, and the point S' coincides with the point C^k. The total input is then consumed in the motor.

PRACTICAL APPLICATION OF THE DIAGRAM.

In the preceding pages, we have derived fully and definitely all characteristics chiefly from theoretical considerations. It remains to show the agreement between the theory and practical tests. If we have the completed motor before us, we need the following data in order to determine its properties.

1. Ratio of the

$$\frac{\text{Reluctance of leakage path}}{\text{Reluctance of path of main field.}}$$

2. Resistance of the circuits.
3. Iron and friction losses.
4. Current, motor running light.
5. Current, motor blocked.

Some of these qualities, as, for instance, the reluctances, cannot be directly measured. As we have determined in the preceding work, it would be possible to measure these reluctances if the losses were negligible, that is, if the electrical resistances were equal to zero. Then, if we measure the primary current, with the secondary circuit open, and also the current with the secondary circuit closed and stationary, we would have, in the first case, the value $A\ C$, and in the second case the value $A\ D$ and

$$\frac{A\ D}{A\ C} = \frac{\text{reluctance of leakage path}}{\text{reluctance of path of main field}}$$

As shown, in consequence of the losses, the actual limiting points are C° and C^k, instead of C and D.

These points are easily obtained, and from them the centre of the circle O_c. We then find the points C and D, and all other characteristic points in a similar manner. It is only necessary to perform two tests with ammeter and wattmeter.

(1) Motor running without load. The current $A\ C^o$ and the angle of phase difference $\theta = C^o\ A\ B$.

(2) With stationary and short-circuited rotor. The current $A\ C^k$ and the angle of phase difference $\theta = C^k\ A\ B$.

If it is impossible to allow full short-circuit current to flow, it is sufficient to make the test at a lower impressed electromotive force. The real short-circuit current is then proportional to the electromotive force applied. If we represent the electromotive force in direction by the line $A\ B$; the direction and magnitude of the no load current by $A\ C^o$, and the direction and magnitude of the short-circuit current by $A\ C^k$, then the center O_c of the circle is determined, since it must lie on a line at right angles to $A\ B$. If, by a third test, the resistance R_1 of the primary circuit is determined, the point E^k can be found by making $C^k\ E^k$ proportional to $I_1\ R_1$. Thence we find the centers O_E and O_F and the slip line $s\ C^k$.

These two simple tests, one at no load, and one with short-circuited and stationary rotor, give sufficient data to indicate clearly all characteristic points, and to construct the load curves. Since the only tests necessary are on the motor running light, and with the rotor blocked, the method renders any exact load tests unnecessary. Moreover, the diagram gives at a glance the reasons for the various characteristics of the motor, and shows how the proportions need to be changed to obtain any desired result.

For instance, in a motor with low electrical losses,

the line $E^k D$ which represents them will be very small. Therefore, the point E^k will be very high, and the starting torque $E^k e^k$ will be small. As is well known, this may be avoided by inserting, by means of slip rings, resistances in the secondary circuits.

If it is desired to avoid slip rings, the resistance of the short-circuited secondary must not be made too small. Then the points E^k and C^k are removed farther from D. Moreover, the variation of the slip line $S\,C^k$ and the slip is greater.

The motor will have, under load, greater speed variations and low efficiency.

It is possible, however, to obtain good torque without excessive losses, if the motor is so designed that the normal load is reached at a point far beyond C', that is, if the field is made strong. But in this case the magnetizing current is very large in proportion to the energy current and cos θ will be unfavorably small. Such a motor will, moreover, be comparatively large. The size of the motor is, in general, determined by the maximum abscissa $O_c\,C'''$, The abscissæ for normal load $c'\,C'$ is then chosen so that the desired relation between normal load and maximum load is obtained. The leakage constant of Rothert, which fixes for each overload capacity a certain ratio between current and field, corresponds with this point of view.

In the diagram this relation is expressed by the ratio $\frac{C\,C'}{C'\,D}$, that is by the tangent of the angle $C'\,D\,C$. If this angle becomes 45°, the motor has its maximum energy input. The smaller this angle, the larger the maximum capacity of the motor. Rothert does not consider in his constant the magnetizing current. The overload capacity of a certain motor type is limited by the

magnetizing current, which increases so that the phase difference becomes large and the power factor small.

The magnetizing current is determined by the air-gap. If the air-gap is changed, the magnetizing current, $A\,C$, changes; while the diagram above the point C remains unchanged.

Therefore, we can say with Rothert that, disregarding the magnetizing current, the diagrams for all multi-phase motors are identical.

The maximum input C''' is proportional to the energy current, that is, to the radius of the circle. In case of double input capacity, for instance, the normal input corresponds to one half of the radius.

It is possible to judge of the output of a motor approximately by the short circuit current. Since the latter is nearly equal to the diameter of the circle, we may employ the following rough approximation:

The normal capacity of a motor corresponds to about one-quarter of its short circuit current, that is, if a motor of 100 volts has a short-circuit current of 100 amperes, its normal output corresponds to

$$\tfrac{1}{4}\times 100\times 100\times\sqrt{3} = 4300 \text{ watts, or about 5 h.p.,}$$

assuming 75% efficiency, that is, the normal output corresponds to one quarter of the current, and is proportional to one quarter of the current necessary to force the total field through the air-gap.

Obviously, with constant impressed electromotive force, this current varies inversely as the square of the turns, directly with the length of the leakage path, and therefore with the diameter d, and inversely with the width of the effective iron; hence

$$L \propto \frac{d}{b} \times \frac{1}{m^2}$$

For large output, therefore, d must be large and b small.*

These interesting, and extremely simple, considerations become, after a little practice, very serviceable.

Considering that m can be replaced by

$$\frac{\text{electromotive force}}{\phi}$$

and L by electromotive force $\times$ current i, we obtain all values of the Rothert leakage constant, or

$$\frac{m\,i\,b}{\phi\,d}$$

Good design in standard motors requires good efficiency and low heating, and should further fulfil the following conditions: maximum output of one and one-half to two times the normal output; starting torque about equal to normal full load running torque and moderate starting current; small "no load" current, varying from one-third to one-fifth of the normal full load current, depending on the size of the motor; the power factor should be high and the slip small.

The above conditions being more or less conflicting,

* This statement does not agree with the results of modern research made by Behn-Eschenburg and others. It has been shown and generally recognized that the end connection leakage is a considerable part of the total leakage, and that this kind of leakage is large in motors which have a comparatively large diameter and a small iron width; therefore the output of a motor is not always improved by simply making d larger and b smaller. In very many cases, the opposite effect is obtained. (See *Journal* of the Institute of Electrical Engineers, Vol. 33, page 239, April, 1904.) Paper by Behn-Eschenburg on Magnetic Dispersion in Induction Motors.—ROWE-HELLMUND.

it is not always possible to satisfy them all at the same time, especially in small motors.

The derivation of the circle diagram above given, contains a simplification which leads to a slight inaccuracy which did not appear in the original derivation, published in the *Electrotechnische Zeitschrift*, October 11, 1894, to which reference has already been made.

In consequence of the resistance of the primary winding, the

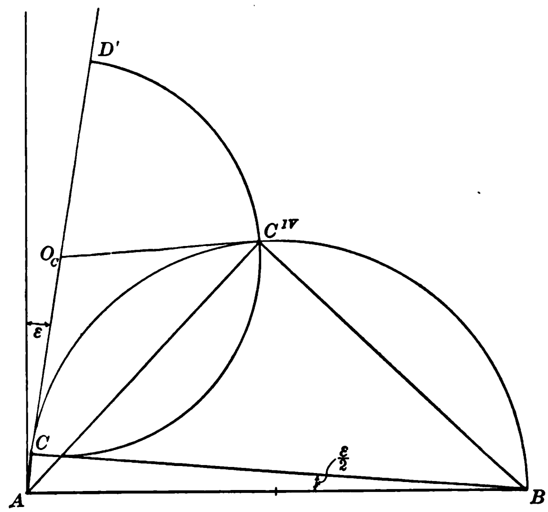


FIG. 8.

theoretical magnetizing current is not entirely wattless, and its phase difference, with respect to the primary impressed electromotive force, is somewhat less than 90°. Thus, the magnetizing current $A\ C$ is not exactly at right angles to $A\ B$, and the center of the circle O_c is not exactly in the perpendicular erected at A on $A\ B$. Practically, the inaccuracy introduced is so small that in most cases it does not introduce a measurable error. In very small motors, only, is the result of the in-

accuracy noticeable, and in these cases, the result is always somewhat better than that shown by the diagrams.

In the first rough laying out of a motor, this refinement will be found unnecessary, inasmuch as the use of the uncorrected diagram introduces only a factor of safety, while in cases of relatively high stator resistance the correction is easily made.

For the sake of completeness, the method of applying the correction will be given: The original circle diagram on page 563, E.T.Z., 1894, had the form shown in Fig. 8. The influence of the primary resistance, in causing a shifting of the line $A\ D'$, representing the magnetizing current, has been much exaggerated in the figure.

The theoretically exact position for the center of the circle O_c is determined as follows: The point C for the theoretical no load condition (assuming no friction and no core losses), as well as the point C^{iv} for the theoretical short circuit condition (the resistance of secondary being equal to zero), must be situated on a half circle, the diameter of which is the impressed electromotive force $A\ B$. The center of the circle O_c is on the intersection of the tangent to the semi-circle $A\ B$ at C and C^{iv}.

The center of the circle O_c is not, therefore, exactly on the perpendicular to $A\ B$, but on a line inclined to it by an angle ε. This angle is

$$\angle\ \varepsilon = 2\ C\ B\ A, \text{ and } \sin C\ B\ A = \frac{A\ C}{A\ B}$$

In motors, as usually constructed, this ratio representing the relation between the drop of potential caused by the magnetizing current and the impressed electromotive force, is so small that the result is not influenced by a measurable amount. Therefore, we shall use the approximate diagram in the following examples illustrating the application of the diagram.

APPLICATION OF THE DIAGRAM.

It may be well to exemplify the preceding considerations by applying them to several practical cases. The motors considered in the following were built by the "Societe Electricité et Hydraulique" and were intended to give a starting torque equal to twice the full load running torque.

Two Horse Power Motor:

1. At no load, 120 volts applied.

    4.1 amperes,
    210 watts,

2. Short-circuited secondary and stationary rotor, 120 v. applied.

    55 amperes,
    6940 watts,

3. Resistance of primary member per phase.

    .344 ohm.

From the foregoing data, the diagram (Fig. 9) was constructed.

From the watts at no load, we obtain the energy current:

$$I_\omega = \frac{210}{120\sqrt{3}} = 1 \text{ ampere.}$$

First construct the no load current triangle $A\ C\ C^\circ$, in which

$A\ C^\circ$	= no load current	= 4.1	amperes
$C\ C^\circ$	= energy current	= 1	"
$A\ C$	= magnetizing current	= 4	"

The energy current $C\ C^\circ$ represents the no load losses by friction, hysteresis and eddy currents. These losses are assumed to be constant for all loads. Therefore, they must be allowed for by subtracting a constant amount from the mechanical output and torque. This can be accomplished in the diagram by drawing a line $x\ y$ through C° parallel to $A\ C$ and measuring the output and torque from the line $x\ y$. At the point C', for instance, the input is $C'\ c'$, the torque $E'\ e_1'$, and the output $f_1'\ F'$.

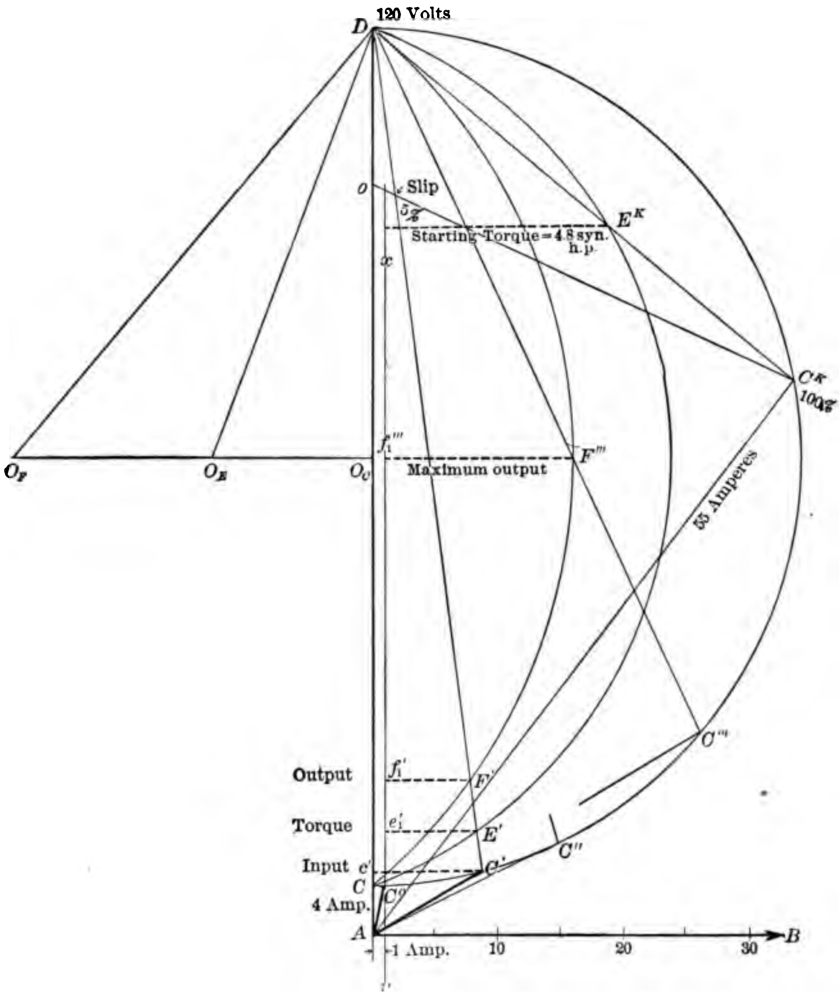

120 Volts
D
o
Slip
5%
E^K
Starting Torque = 4.8 syn. h.p.
x
C^K
100%
f_1'''
F'''
O_F
O_E
O_C
Maximum output
55 Amperes
C'''
Output
f_1'
F'
Torque
e_1'
E'
C''
Input c'
C'
C
C^0
4 Amp.
A
B
1 Amp.
10
20
30

Fig. 9.

From the stationary test we obtain an energy current

$$\frac{6940}{120\sqrt{3}} = 33.5 \text{ amperes, or—}$$

$$\cos\theta = \frac{33.5}{55} = .61$$

This value enables us to determine the point C^k, since $A\,C^k$ is proportional to the current with rotor blocked, or 55 amperes, and cos $C^k\,A\,R$ equals the power factor, or .61.

From these points C° and C^k, it is easy to find the center O_c, and the corresponding circle $C\,C^\circ\,C^k\,D$.

If now the line $C^k\,D$ is drawn, then the circle giving the mechanical output is determined, since we know that the output of the short circuited and stationary rotor corresponding to the point C^k is zero. Therefore, the intersection of the line $C^k\,D$ with the circle must fall at the point D. The line $C^k\,D$ must therefore be tangent to the output circle at D. Hence, the center O_E and the circle E' itself is determined.

As we have seen, the mechanical output derived from this circle must be decreased by an amount equal to the no load losses which are represented by the parallel line $x\,y$, or, in other words, the output is $E'\,e_1$, etc.

Finally, in order to find the third circle for the torque, we must remember that the line $C^k\,D$ represents the electrical losses of locked rotor. These losses divide themselves into two parts, primary losses and secondary losses, which are proportional, respectively, to the reduced resistance in each member.

Since the total field $A\,D$ corresponds to an electro-

motive force of 120 volts, we find in the same scale the loss vector at short circuit.

$$C^k D \times \frac{120}{A\,D} = 73 \text{ volts.}$$

In the primary member, of which the resistance per phase is .344 ohm, and in which the short circuit current is 55 amperes, the drop of potential is

$$\sqrt{3} \times 55 \times .344 = 32.7 \text{ volts.}$$

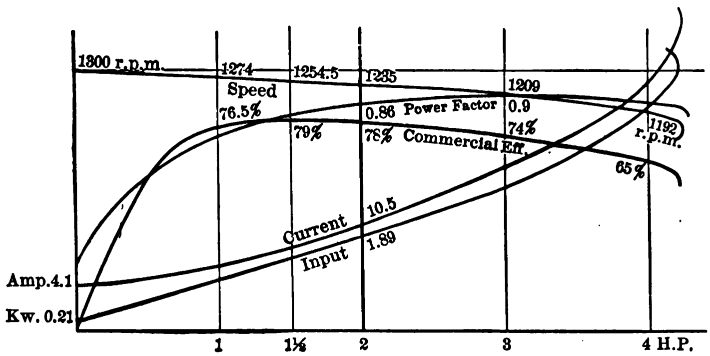


FIG. 10.

If we therefore make $C^k E^k$ equal to $C^k D$ multiplied by $\frac{32.7}{73}$, we know that the torque circle must pass through the point E^k, and therefore this circle, and its center O_E are determined.

Finally, it remains to draw the line $C^k s$ from C^k at right angles to $O_E D$, from which may be found the slip for any load. At no load the slip is zero, and at full load 5%.

The diagram now shows all the characteristic properties of the motor. It is possible to transfer, by means

of a compass, all of the various values directly to a rectangular system, as shown in Fig. 10.

The normal output of the motor is two horse power at 78% efficiency, 86% power factor and 5% slip. The maximum output is more than double the normal full load value. The maximum power factor is .90. Finally, the distance between C^k and the line $X\ Y$ gives

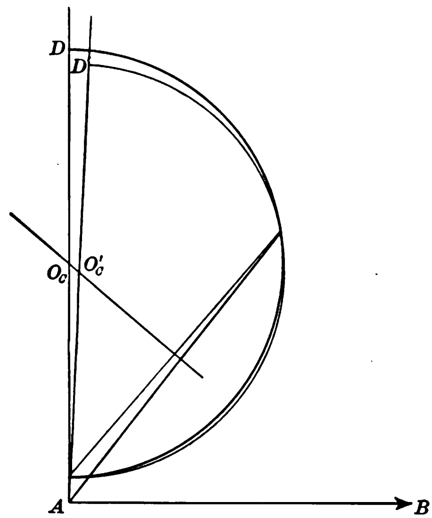


FIG. 11.

a starting torque of four synchronous horse power, the starting current being 55 amperes. The motor thus meets the requirements usually assumed.

The above motor is a small one; in the diagram for which the center O_c should be shifted from the line at right angles to $A\ B$, as was shown in the note on page 16, it may be here shown that this correction is very small and that it may be practically neglected.

We have:

Impressed electromotive force..............	120 volts
Magnetizing current........................	4 amperes
Resistance of stator winding per phase.......	.344 ohm

and therefore

$$\operatorname{Sin}\frac{\varepsilon}{2} = \frac{\sqrt{3} \times .344 \times 4}{120} = .02$$

The angle ε is shown in Fig. 11, and the dotted circle shows the corrected position of the circle.

If one calculates results from this corrected circle, it will be seen that the maximum power factor has increased from .9 to .91, and that other values, such as efficiency and maximum output, are unchanged. The difference in power factor is smaller than can be observed with ordinary measuring instruments.

As a matter of fact, only in very small low speed motors is a difference noticeable, and in these cases the correction may be made. Exact calculations are anyhow difficult in the case of small motors, and since we know that the actual values are somewhat better than those given by the diagram, it is hardly necessary to make a correction.

SEVEN HORSE POWER MOTOR.

Tests on a seven horse power motor gave the following data:

1. At no load, 110 volts per phase.
   I = 6.3 amperes per phase,
   A = 100 watts per phase.
2. Locked and short-circuited rotor, 110 volts per phase.
   I = 118 amperes per phase,
   A = 6670 watts per phase.
3. Resistance per phase = .142 ohm.

From these results, the diagram shown in Fig. 12 was constructed.

The no load energy current is

$$I_\omega = \frac{100}{110} = .91 \text{ ampere.}$$

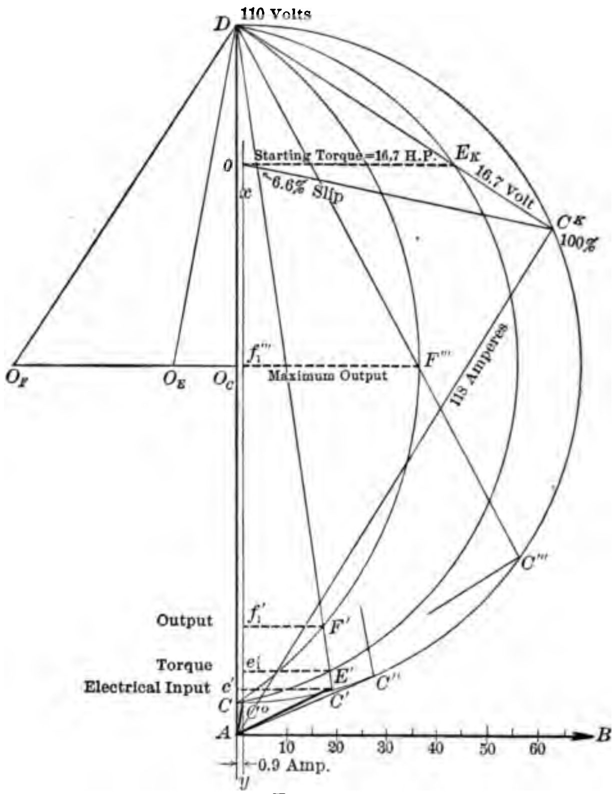


FIG. 12.

Draw the current triangle $A\ C\ C°$, in which

$A\ C°$ = no load current = 6.3 amperes

$C\ C°$ = energy current = .91 "

$A\ C$ = magnetizing current = 6.2 amperes.

For the no load losses, draw the line $H\,Y$, parallel to $A\,C$, passing through the point C°. The mechanical output, as we know, is measured from the line $h\,y$, since the constant no load losses have always to be subtracted. The test of the stationary and short-circuited rotor, gives us the energy current,

$$\frac{6670}{110} = 60.7 \text{ amperes} = i \cos\theta \text{ or}$$

$$\cos\theta = \frac{60.7}{118} = .515 \text{ power factor.}$$

Thus we find the point C^k of the diagram. The center O_B of the input circle can now be easily found by construction. The point O_F is obtained by drawing through D a perpendicular to $C^k\,D$ until it intersects the line $O_B\,O_F$ in the desired point O_F. In other words, the line $D\,C^k$ is a tangent to the circle of mechanical output. It now remains to find the third, or torque circle. This is done by dividing the line $C^k\,D$ which represents the electrical losses at short circuit in the ratio of the resistances of the primary and secondary circuits. The third circle then passes through this point.

In the diagram, the field per phase corresponds to a potential of 110 volts, and the loss vector at short circuit, measured in the same scale, is

$$\frac{C^k\,D \times 110}{A\,D} = 57 \text{ volts.}$$

If the primary winding, which has a resistance of .142 ohms per phase, we have 118 amperes at short circuit, therefore the drop of potential is $118 \times .142 = 16.8$ volts.

To find the intersection of the third circle with $C^k\,D$,

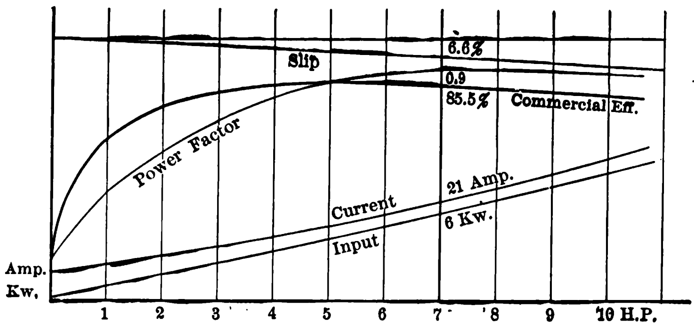

6.6%
Slip
0.9
85.5%
Commercial Eff.
Power Factor
21 Amp.
Current
6 Kw.
Input
Amp.
Kw.
1
2
3
4
5
6
7
8
9
10 H.P.

FIG. 13.

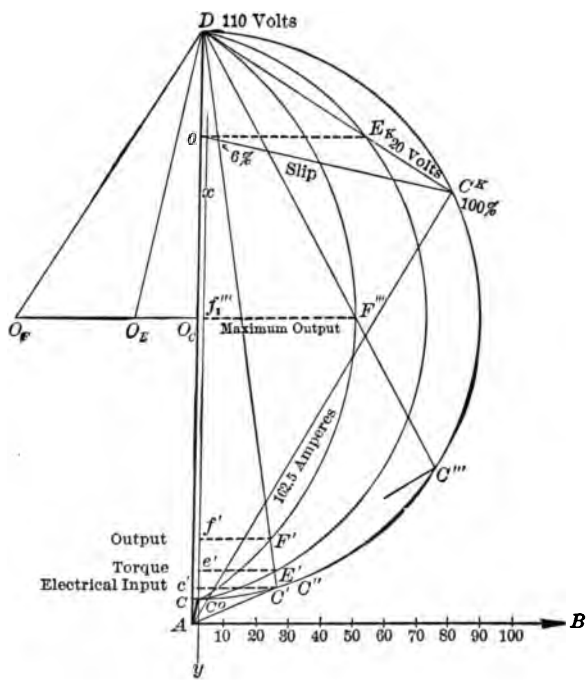

D 110 Volts
O
6%
Slip
E_K
20 Volts
C^K
100%
x
f_1'''
F'''
Maximum Output
O_F
O_E
O_C
162.5 Amperes
C'''
Output
Torque
Electrical Input
f'
F'
e'
E'
c'
C' C''
C
C^0
A
10 20 30 40 50 60 70 80 90 100
B
y

FIG. 14.

we must lay off this value $= C^k E^k$ on $C^k D$ in the ratio of 16.8 to 57.

Finally, erect a perpendicular from C^k on the line $O_E D$ and obtain the slip for any load. At no load the slip is very small; at full load only 6.6%.

Fig. 13 shows the characteristic curves of the motor as functions of the output. The full load output is seven horse power. At this load, the efficiency is

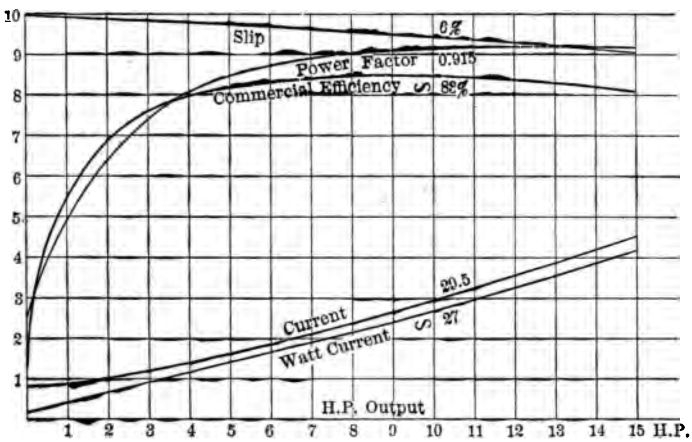


FIG. 15.

85.5%, and the power factor 90%. The power factor reaches a maximum of about 91% at a load of about nine horse power. The motor starts with a torque of 16.7 synchronous horse-power, and has a maximum output of 15.4 horse-power.

TEN HORSE POWER MOTOR.

Figs. 14 and 15 give the phase diagram and the characteristic curves of a 10 horse power motor.

The tests gave the following results:

1. At no load, 110 volts applied per phase.
   I = 8 amperes,
   A = 197 watts.
2. Rotor locked and short-circuited.
   110 volts applied.
   I = 162.5 amperes,
   A = 9 kw.
3. Resistance per phase = .123 ohm.

Fig. 14 is the diagram constructed from these data. Fig. 15 shows the characteristic curves. The following results are obtained at full load of 10 horse power:

Efficiency	= 83%
Current	= 29.5 amperes
Electrical Input	= 2.95 kw.
Power Factor	= .915
Slip	= 6%
Maximum output	= 21 horse power.

THE INDUCTION MOTOR AS GENERATOR.

It is a well known fact that an induction motor, if driven above synchronism, will act as a generator analagously to the direct current shunt machine. It is interesting that the circle diagram gives also light on this use of the motor, if the circles are completed on the left hand side of the line A D as in Fig. 16. The slip is now negative, and what was before mechanical output, is now mechanical input. Further, the electrical input now becomes the electrical output.

We see that the generator gives now exactly the same current for the same phase difference, as when employed as a motor. In other words, the electrical energy is the same in both cases. The mechanical input is, of course, considerably larger in the case of the gen-

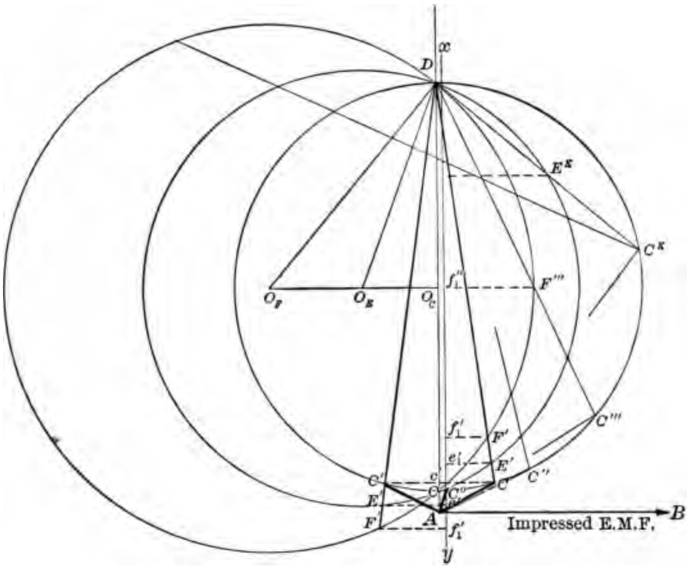

x
D
E^x
C^x
O_F
O_E
O_C
f_1''
F'''
C'''
f_1'
F'
e_1
E'
C''
C'
c
E
F
A
f_1'
y
Impressed E.M.F.
B

FIG. 16.

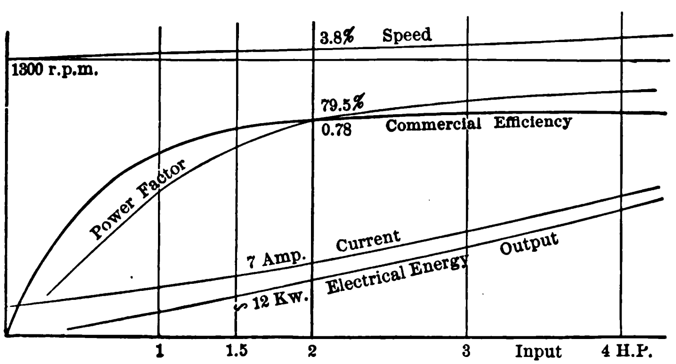

3.8%
Speed
1300 r.p.m.
79.5%
0.78
Commercial Efficiency
Power Factor
7 Amp.
Current
12 Kw.
Electrical Energy
Output
1
1.5
2
3
Input
4 H.P.

FIG. 17.

erator than the mechanical output of the motor. It is easily seen that the difference amounts to twice the losses, since now all losses add themselves to the electrical output, while in the motor they were subtracted from the electrical input.

From a theoretical standpoint, the curves become exceedingly interesting if drawn, not as functions to the mechanical output, but as functions to the electrical input; that is, as functions of the values which correspond to the energy currents which are the abscissae of the diagram (Fig. 18).

The curves thus obtained are of regular mathematical forms, and are simple closed curves. The curves of torque and mechanical output are ellipses. The power factor curve is a function of an angle, etc.

There are three essentially different parts of the curves to be considered:

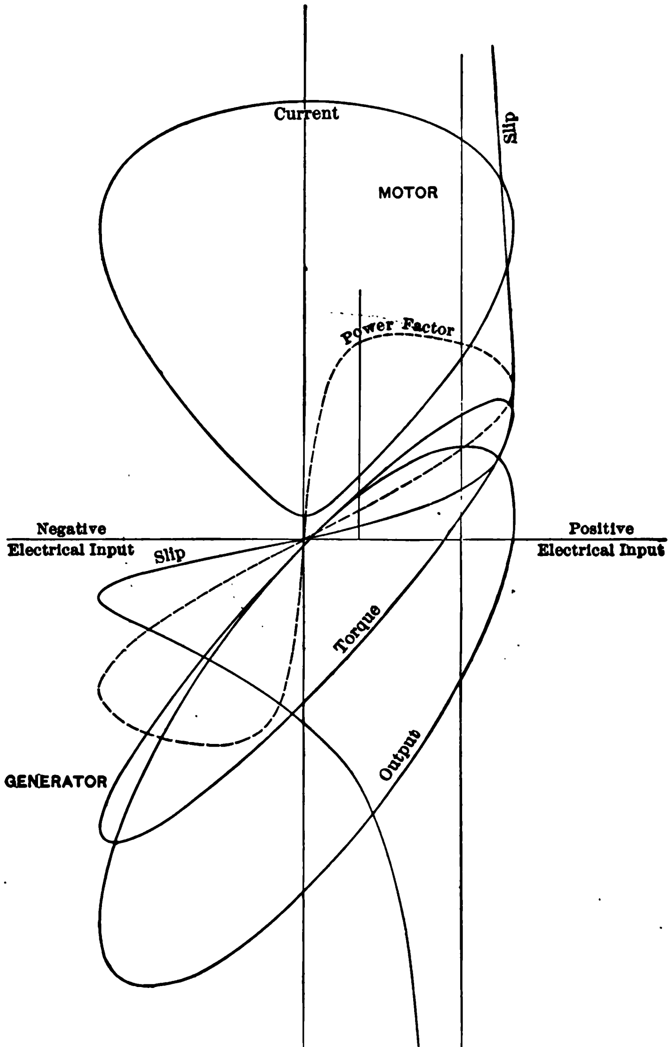
1. As a motor, (slip 0 to 1) $C_0 \rightarrow C^k$.

$$\left.\begin{array}{ll} \text{2. As a generator} & \left\{\text{slip} < 0\right. \\ \text{3. As a generator} & \left(\text{slip} > 1\right. \end{array}\right\} C_0 \rightarrow D \rightarrow C^k.$$

With these general diagrams and curves, all essential points of the use of the present method are exhausted. All complicated details of the motor are based on two phenomena, magnetic leakage and electric resistance. These are essentially the only values which determine the operation of the motor, and it remains for the designer to choose the proper proportions.

SINGLE PHASE MOTORS.

The fundamental theory of the polyphase motor is much simplified by the introduction of the principle of the rotating field, but the theory of the single-phase motor is somewhat more difficult.


Current
Slip
MOTOR
Power Factor
Negative
Electrical Input
Slip
Positive
Electrical Input
Torque
Output
GENERATOR

The single-phase motor possesses a single phase winding for a simple alternating current (Fig. 19), and can therefore produce a simple alternating field in the direction of the coil axis only. Such a field, of course, will not cause rotation of a short-circuited armature, without commutator or similar devices. Therefore it seems, at first thought, somewhat astonishing, that such a motor can operate at all, and indeed, in its

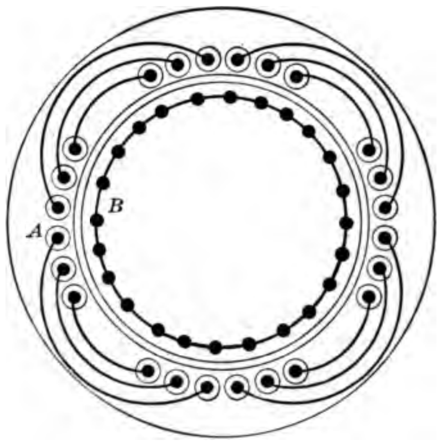


FIG. 19.

simplest form such a motor will not start; however, after the motor has been started by any means whatsoever, it then operates exactly like a polyphase motor.

This phenomenon is explained by the fact that in the single phase motor, rotating at a certain speed, there is a field caused in the secondary which rotates with it. Between this field and the ampere turns of the single phase winding, a torque is established in the same manner as between the field of a short-circuited second-

ary and one of the two or three phase windings of the polyphase motor. The torque, however, is pulsating, since there is no second phase, and it becomes a maximum, falls to zero and again reaches a maximum varying with the alternations of the field.

In the case of induction motors, single as well as polyphase, we may consider the short-circuited secondary as a field magnet of a synchronous motor. The only difference is that the field does not rotate in the induction motor in synchronism with the armature, but has a small relative rotation with respect to the armature, corresponding to the slip.

In the short-circuited secondary of a single-phase motor there exists, just as in the polyphase motor, a constant rotating field notwithstanding the fact that the current exciting winding produces only a simple alternating field. This is explained by the characteristic property of the short-circuited winding. We know that any short-circuited winding tends strongly to keep up the field interlinked with it, since any change in this field causes current in the short-circuited winding which opposes such change. This counteracting effect is the larger the lower the resistance of the winding and the greater the frequency.

If we consider the secondary of a single phase motor revolving synchronously, the primary alternating field will not appear as an alternating field in the secondary, since in the same time, during which the field intensity starting from zero reaches a maximum and decreases again to zero, the secondary has turned through 180°, and the primary alternating field will therefore appear as a pulsating field, and will have always the same direction in regard to the secondary member.

Each pulsation of the secondary field causes, how-

ever, in the secondary winding wattless currents which diminish the pulsation to a negligible minimum, *i.e.*, they keep the pulsating field of the short-circuited secondary practically constant. In the positions where the field corresponding to the exciting current tends to exceed this constant value, the secondary ampere turns demagnetize. In other positions they furnish the magnetizing current.

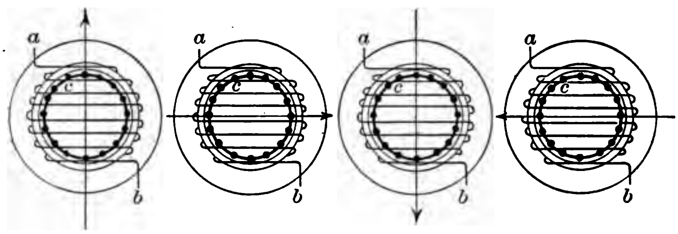


FIG. 20. FIG. 21. FIG. 22. FIG. 23.

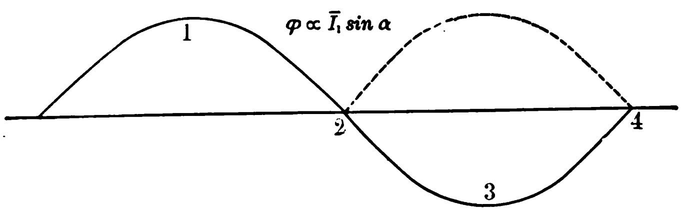


FIG. 24.

In order to obtain a still clearer understanding of this phenomena, let us consider the four characteristic positions:

In Figs. 20–23, let $a\, b$ be the primary winding, and c the secondary, which rotates synchronously.

The field caused by the current I_1 will then be as shown in Fig. 24.

In the positions 1 and 3, the maximum value of the current acts once in one direction and once in the other. Since, however, the rotor has meanwhile turned through 180°, its field will have in the positions 1 and 3 the same direction, and will have a characteristic shown by the dotted line. The above described action of the rotor now appears, and prevents the field from

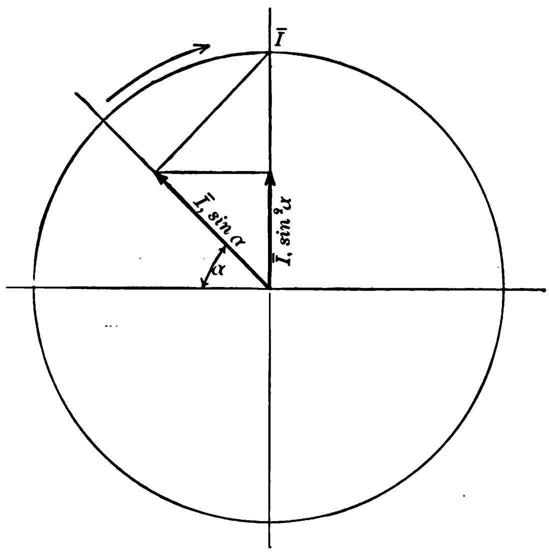


FIG. 25.

being strongly pulsating, the pulsations being smoothed out. Therefore, we obtain a constant field rotating with the rotor.

It is now possible to picture the action of the rotor as follows: Let I_1 be the magnetizing ampere turns of the exciting member, I_2 those of the rotor; then these two obviously must combine to form the ampere turns

I_1 which produce the constant field. This is true, if, for the moment, we neglect the leakage.

If we assume again that the alternating current is a sine function, and that $\overline{I}_1$ is the maximum value of this current, the motor being unloaded, then the instantaneous value of the ampere turns of the exciting member at no load is proportional to

$$I_1 \propto \overline{I}_1 \sin \alpha$$

If we consider the short-circuited secondary to be stationary, and the exciting winding to rotate, then the constant field must also be stationary. This field is produced by the ampere turns of the rotating exciting member, and these ampere turns vary according to a sine law. Therefore, the field of the short-circuited secondary must have a component proportional to

$$I_1 \sin \alpha \propto \overline{I}_1 \sin^2 \alpha \propto \overline{I}_1 \; \frac{1 - 2 \cos \alpha}{2}$$

The field of the short-circuited secondary must, however, be constant, and therefore the resulting ampere turns must be proportional to $\overline{I}$, where $\overline{I}$ is the magnetizing current. The difference between the magnetizing ampere turns and those active in the exciting member must, therefore, be produced in the short-circuited secondary.

Thus, if $\overline{I}_2$ is the value of the current in the secondary we have

$$\overline{I}_2 = \overline{I} - \frac{\overline{I}_1}{2} \; (1 - \cos 2 \alpha)$$

$$\overline{I}_2 = \overline{I} - \frac{I_1}{2} + \frac{I_1}{2} \; \cos 2 \alpha$$

The currents in the secondary can only be periodic functions of the time, that is, there can be no constant terms. Thus, we have,

$$\bar{I} = \frac{\bar{I}_1}{2} = \dot{0}$$

$$\bar{I} = \frac{\bar{I}_1}{2}$$

$$\bar{I}_2 - \frac{\bar{I}_1}{2} \cos 2\,\alpha$$

Therefore, the ampere turns of the primary member are—

$$A\ T_1 \propto \frac{\bar{I}_1}{2} - \frac{\bar{I}_1}{2} \cos 2\,\alpha$$

and those of the secondary are—

$$A\ T_2 \propto \frac{\bar{I}_1}{2} \cos 2\,\alpha,$$

and the magnetizing ampere turns

$$A\ T \propto \frac{\bar{I}_1}{2} \infty \bar{I}$$

These equations indicate that in the single-phase motor the magnetizing current I is half of the no load current. The four characteristic positions are now represented in Fig. 26.

The sum of the two fields gives a straight line corresponding to the amplitude $\frac{\bar{I}_1}{2}$.

The single fields are represented by the full and dotted waves. The rotating field is not quite constant, but is slightly pulsating. It is obvious, however, that this may be neglected.

Another approximation previously made will now be corrected. Owing to the presence of leakage, the ratio

$$\frac{\text{No load current}}{\text{Magnetizing current}} \text{ is not exactly two.}$$

In the position of Figs. 27 and 28 the secondary fields are the same. In Fig. 27 it is induced by the

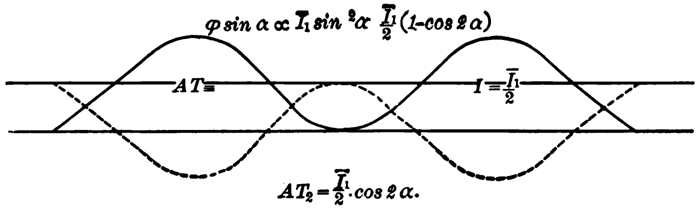


FIG. 26.

primary winding by the magnetizing current $I = I_1 - I_2$, or, introducing the reluctance of the secondary

$$\phi_a \propto \frac{I}{\rho}$$

In the other positions (Fig. 28), the secondary field is produced by the ampere turns of the short-circuited secondary. The reluctance will be smaller, since the field now has two paths, the main path and the leakage path. If we call the reluctance in this case ρ' then

$$\phi_a \propto \frac{I_2}{\rho'}$$

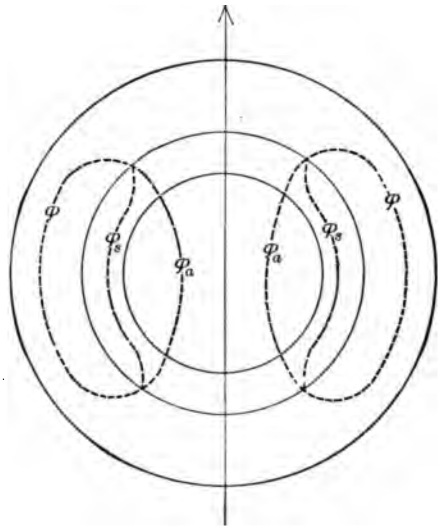

φ
φ_s
φ_a
φ_a
φ_s
φ

FIG. 27.

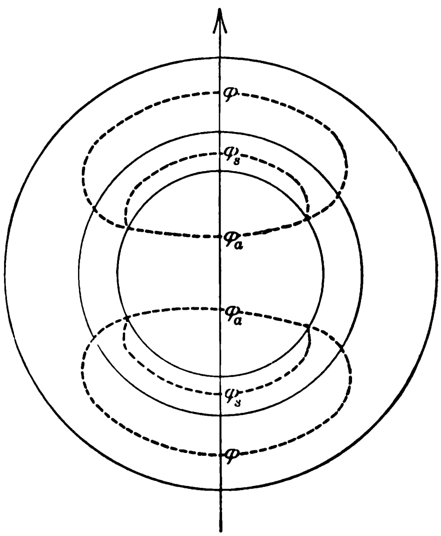

φ
φ_s
φ_a
φ_a
φ_s
φ

FIG. 28.

We have seen that the secondary field remains almost constant, and therefore we have

$$\frac{I}{\rho} = \frac{I_2}{\rho'}, \text{ or, since}$$

$$I_2 = I_1 - I$$

$$\frac{I}{\rho'} + \frac{I}{\rho} = \frac{I_1}{\rho'} \text{ and}$$

$$I\,(\rho+\rho') = I_1\,\rho'$$

$$\frac{I_1}{I} = \frac{\rho+\rho'}{\rho} = \frac{\text{No load current}}{\text{Magnetizing current}}$$

By substitution of the simple reluctances ρ_1, ρ_s and ρ_2 the ratio can appear in a somewhat different form, in which only known values appear. It will be seen then that ρ' is little different from ρ and that the ratio is almost two.

The phenomena in single-phase motors are thus as follows:

In the unloaded single-phase motor, there is formed in the short-circuited secondary, as in the polyphase motor, a field which rotates with the short-circuited secondary. This field when it coincides with the axis of the primary winding is produced by the latter. At right angles to this position, it is produced by the no load currents of the short-circuited secondary. The latter are wattless and are produced by the primary member. Therefore, the no load current equals twice the magnetizing current, or, more accurately, the no load current is $\frac{\rho+\rho'}{\rho}$ times the magnetizing current.